**PROBLEM 10.** In order to fit more columns into a smaller space, the designers realized that they should investigate the possibility of tapering the columns and then "nesting" them for transportation, like a stack of paper cups. Fig. 4.13 illustrates the idea. Under this scheme, each column would be made of two tapered half-columns, with their wider openings joined; half-columns could then be nested for stowage in the cargo bay. Tapered columns have been developed and tested for strength. If \( r_1 \) is the radius of the smaller end and \( r_2 \) the radius of the larger end, tests showed that an optimum taper ratio is \( t = 0.41 \) and that such a tapered column is actually stronger; it can carry about 30 percent more load before buckling than an untapered column of the same weight.

![Fig. 4.13 Tapered Column Concept](image)

**a.** If the mean radius is to be 3.8 cm as before, and \( \frac{r_1}{r_2} = 0.4 \), find the values of \( r_1 \) and \( r_2 \).

**Solution:** We have \( \frac{r_1 + r_2}{2} = 3.8 \) and \( r_1 = 0.4 \times r_2 \). Clearing the fraction and substituting, we get

\[
0.4 \times r_2 + r_2 = 7.6 \\
1.4 \times r_2 = 7.6
\]

\[
r_2 = \frac{7.6}{1.4} = 5.4 \text{ cm}
\]

\[
r_1 = (0.4)(5.4) = 2.2 \text{ cm}
\]

**b.** Fig. 4.14 (a) and (b) display the geometry of the tube nesting, where \( d_1 = 2 \times r_1 \), \( d_2 = 2 \times r_2 \).

\( l \) is the length of a half-column, and \( \Delta \) is the tube-nesting separation. Show that \( \Delta = \frac{2l}{r_2 - r_1} \) and find an expression in terms of \( l \) and \( \Delta \) for the numbers of half-columns that will fit into one stack the length of the Space Shuttle cargo bay.
Solution: In Fig. 4.14(b), if we insert the horizontal line shown and letter some key points as indicated (Fig. 4.15), \(\Delta ABE'\) and \(\Delta BCD\) are similar, so \(AB/BC = BE'/CD\). We have \(AB = \Delta\), \(BC = l\), \(CD = r_2 - r_1\), and we shall approximate \(BE'\) by \(BE = t\). Then, with this approximation and the proportion above,

\[
\Delta = \frac{tl}{r_2 - r_1}
\]

From Fig. 4.14(a), we have one half-column of length \(l\) on the left end, in which we nest \(n\) additional half-columns, where \(n = INT \left[\frac{18.3 - l}{\Delta}\right]\). (INT signifies the greater integer which is less than or equal to the number in the square bracket).

Now the number of half-columns that will fit into one stack is \(N = 1 + n = 1 + INT \left[\frac{18.3 - l}{\Delta}\right]\)

c. For the truss assembly of Problem 9, determine the volume occupied strut columns if they are made of half-columns as described here and nested for stowing in the cargo bay.

Solution: We have \(l = \) half-column length = \((1/2)(10.4) = 5.2\) m

\[
\Delta = \frac{tl}{r_2 - r_1} = \frac{(0.57\times10^{-3})(5.2)}{(5.2 - 2.2)\times10^{-2}}m = 9\times10^{-2}m
\]

\[
N = 1 + INT \left[\frac{18.3 - 5.2}{9\times10^{-2}}\right] = 1 + INT \left[145.6\right] = 1 + 145 = 146
\]

We had a total of 2268 columns, or 4536 half-columns, so this means there well be \(INT \left[\frac{4536}{146}\right] = 31\) stacks, and one additional shorter stack.

Each stack is 18.3 m long (although one stack will be shorter) and has a radius of 5.4 cm, so its volume is \(\pi (0.054)^2 (18.3) = 0.17\) m\(^3\). The total volume of the 32 stacks is a little less than \(32 \times 0.17 = 5.4\) m\(^3\).

By the analysis in the last part of Problem 9, these stacks will take up \(\frac{5.4}{0.88} = 6.2\) m\(^3\) of space in the cargo bay, and now the materials for the truss assembly and the antenna “dishes” can all be transported in a single Shuttle trip.